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An efficient method of analysis of heat transfer during plane strain upsetting of a viscoplastic strip

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A solution for the transient heat transfer during plane strain upsetting of a viscoplastic strip has been developed using a Lagrangian coordinate system and the Green's function available in the literature. A general approximate solution for stress and velocity found elsewhere has been used. In contrast to conventional viscoplastic models adopted in theoretical analyses of metal forming processes, the model considered in the present paper includes a saturation stress. This model used in the conjunction with the maximum friction law predicts the localization of plastic deformation in the vicinity of the friction surface. In turn, localized plastic deformation and temperature are responsible for the generation of a very narrow layer with drastically modified microstructure near the friction surface. The appearance of such layers is reported in numerous experimental works.

KEYWORDS

friction, localization of plastic deformation, singularity, upsetting, viscoplasticity

1 | INTRODUCTION

Numerous experimental studies demonstrate that a very narrow layer with drastically modified microstructure is generated in the vicinity of frictional interfaces in machining and deformation processes (see [1–4] among many others). Plastic deformation is one of the main contributory mechanisms responsible for the generation of such layers.^[5] There are several rigid plastic models that predict highly localized plastic deformation near maximum friction surfaces showing qualitative agreement with the aforementioned experimental results.^[6–9] However, these models do not account for heat generation whereas temperature is another important contributory mechanism responsible for the generation of the narrow layer with drastically modified microstructure near frictional interfaces.^[5] The capacity of the models^[6–9] to predict highly localized plastic deformation comes at a cost: the solutions are singular (some components of the strain rate tensor approach infinity near the friction surface). This causes numerical difficulties. In particular, finite element solutions do not converge.^[10,11] Moreover, it is evident that the singularity in velocity field results in the singularity in the corresponding temperature field since the plastic work rate is involved in the heat conduction equation. Therefore, special methods are needed for the determination of temperature fields in the vicinity of frictional interfaces in metal forming processes with a high accuracy. The present paper concerns with such a method for the process of plane strain upsetting of a viscoplastic strip between two parallel plates.

A property of conventional viscoplastic models that are usually adopted in theoretical analyses of metal forming processes is that the equivalent stress approaches infinity when the equivalent strain rate approaches infinity (see, for example, [12]). In this case it is always possible to find a solution satisfying the sticking boundary condition at friction surfaces.^[13] The behavior of

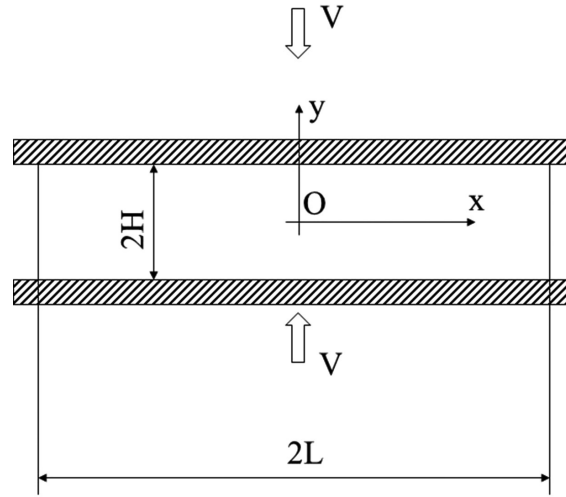


FIGURE 1 Configuration of the problem and Cartesian coordinate system

solutions in the vicinity of friction surfaces qualitatively changes if a saturation stress is included in the model.^[8,14] The physical meaning of the saturation stress is that the equivalent stress approaches to its value as the equivalent strain rate approaches infinity. For such models, the maximum friction law requires that the friction stress at sliding is equal to the maximum possible shear stress supported by the material. It is worthy of note that, in contrast to models with no saturation stress, no solution at sticking may exist under certain conditions. Solutions at sliding are singular in the vicinity of maximum friction surfaces and predict highly localized plastic deformation near such surfaces. It has been shown in [14] that the presence of the saturation stress in the model has a negligible effect of the behaviour of solutions outside a very narrow layer near the friction surface. Therefore, using a model with a saturation stress does not contradict results of conventional tests from which it is impossible to make any definitive conclusion concerning the existence of the saturation stress. Moreover, the conditions under which the material is being deformed within the narrow sub-surface layer are completely different from that encountered in conventional material tests. Therefore, the latter cannot be used to determine the flow stress and other constitutive equations within the layer.^[15]

The process of deformation considered in the present paper is plane strain upsetting of a strip between two parallel plates. A general viscoplastic solution for this process has been developed in [16]. This solution is used in the present paper in conjunction with the dependence between the equivalent stress and equivalent strain rate proposed in [17] to find the distribution of stress and velocity. Using this solution the heat conduction equation is written in Lagrangian coordinates. In this case, the original initial/boundary value problem reduces to one of the standard initial/boundary value problems for the nonhomogeneous heat conduction equation. Therefore, the Green's functions are available in the literature.^[18] An example is presented to illustrate the general solution.

2 | FORMULATION AND SOLUTION FOR STRESS AND VELOCITY

Consider upsetting of a viscoplastic strip between two parallel plates under plane strain conditions. The Cartesian coordinate system (x, y) is shown in Figure 1 for the strip of instantaneous width $2L$ and instantaneous thickness $2H$. The initial values of L and H are denoted by L_0 and H_0 , respectively. The speed of each plate is V . The process is symmetric relative to the axes $y = 0$ and $x = 0$. Therefore, it is sufficient to consider the domain $y \geq 0$ and $x \geq 0$. Let σ_{xx} , σ_{yy} and σ_{xy} be the stress components relative to the Cartesian coordinate system. The direction of flow (Figure 1) dictates that $\sigma_{xy} \leq 0$. The stress boundary conditions are

$$\sigma_{xy} = 0 \quad (1)$$

for $y = 0$, $x = 0$, and $x = L$,

$$\sigma_{xx} = 0 \quad (2)$$

for $x = L$, and the friction law

$$\sigma_{xy} = -\tau_f \quad (3)$$

for $y = H$. Here τ_f is the friction stress at sliding. Let v_x and v_y be the velocity components relative to the Cartesian coordinate system. The velocity boundary conditions are

$$v_y = 0 \quad (4)$$

for $y = 0$,

$$v_y = -V \quad (5)$$

for $y = H$, and

$$v_x = 0 \quad (6)$$

for $x = 0$.

The elastic portion of strain rate is neglected. The equivalent stress σ_{eq} and the equivalent strain rate ϵ_{eq} are defined as

$$\sigma_{eq} = \frac{\sqrt{3}}{2} \sqrt{(\sigma_{xx} - \sigma_{yy})^2 + 4\sigma_{xy}^2}, \quad \epsilon_{eq} = \sqrt{\frac{2}{3}} \sqrt{\epsilon_{xx}^2 + \epsilon_{yy}^2 + 2\epsilon_{xy}^2} \quad (7)$$

where ϵ_{xx} , ϵ_{yy} and ϵ_{xy} are the strain rate components relative to the Cartesian coordinate system. The other strain rate components vanish. The yield criterion is assumed to be

$$\sigma_{eq} = \sigma_0 f\left(\frac{\epsilon_{eq}}{\epsilon_0}\right) \quad (8)$$

where $f(0) = 1$, $df/d\epsilon_{eq} \geq 0$ for all ϵ_{eq} , σ_0 is the yield stress in uniaxial tension at $\epsilon_{eq} = 0$ and ϵ_0 is a reference strain rate. A general solution to the system of equations comprising the equilibrium equations, the yield criterion (8) and its associated flow rule has been developed in [16]. Then, it has been shown that arbitrary functions and constants involved in this solution can be chosen such that the boundary conditions (1) at $y = 0$, (4) and (5) are exactly satisfied. The boundary condition (3) is exactly satisfied if τ_f is a fraction of the bulk shear yield stress. The boundary condition (1) at $x = 0$ and $x = L$ is ignored. The exact boundary condition (2) is replaced with the following approximate condition:

$$\int_0^H \sigma_{xx} dy = 0. \quad (9)$$

Here σ_{xx} is understood to be calculated at $x = L$. The exact boundary condition (6) is replaced with the following approximate condition:

$$\int_0^H v_x dy = 0 \quad (10)$$

Here v_x is understood to be calculated at $x = 0$. Those are typical approximations in the case $H/L \ll 1$.^[19] In [16], the general solution has been adopted to find the distribution of stress and velocity for a Herschel-Bulkley solid. In this case, $\sigma_{eq} \rightarrow \infty$ as $\epsilon_{eq} \rightarrow \infty$ and no solution exists if τ_f is equal to the bulk shear yield stress. The reason for the non-existence of the solution is that the general solution requires the regime of sliding at the friction surface whereas this regime is impossible if τ_f is equal to the bulk shear yield stress and $\sigma_{eq} \rightarrow \infty$ as $\epsilon_{eq} \rightarrow \infty$.^[13] In general, the regime of sliding is possible if $\sigma_{eq} \rightarrow \sigma_s < \infty$ as $\epsilon_{eq} \rightarrow \infty$ where σ_s is the saturation stress.^[8,14] The general solution developed in [16] can be used in conjunction with a viscoplastic model satisfying the latter condition. In particular, based on a great number of independent experimental results for several metals and metallic alloys it has been found in [17] that

$$f\left(\frac{\epsilon_{eq}}{\epsilon_0}\right) = \frac{1 + t(\epsilon_{eq}/\epsilon_0)^m}{1 + (\epsilon_{eq}/\epsilon_0)^m} \quad (11)$$

where $t = \sigma_s/\sigma_0$ and $m > 0$. The boundary condition (3) represents the maximum friction law for this model if $\tau_f = \sigma_s/\sqrt{3}$. Therefore, this boundary condition becomes

$$\sigma_{xy} = -\frac{\sigma_s}{\sqrt{3}} \quad (12)$$

for $y = H$. The regime of sliding at the maximum friction surface is possible if $m \geq 2$.^[8] In what follows, it is assumed that this inequality is satisfied.

It has been found in [16] that the equivalent strain rate is given by (in our nomenclature)

$$\epsilon_{eq} = \frac{2V}{\sqrt{3}H} \left[1 - \frac{\sigma_s^2 y^2}{\sigma_{eq}^2 H^2} \right]^{-1/2} \quad (13)$$

and the velocity components by

$$v_y = -\frac{V}{H}y, \quad v_x = \frac{V}{H}x - \frac{2V}{H} \int_H^y \cot(2\varphi) d\mu + VC. \quad (14)$$

Here C is constant of integration, μ is a dummy variable of integration and

$$\cos(2\varphi) = -\frac{\sigma_s}{\sigma_{eq}} \frac{y}{H}. \quad (15)$$

It is evident from (7), (12) and (13) that $\epsilon_{eq} \rightarrow \infty$ as $y \rightarrow H$. It is seen from (14) that

$$Y = \frac{y}{H} \quad (16)$$

is the Lagrangian coordinate satisfying the condition $YH_0 = y$ at $H = H_0$. Since $\epsilon_{eq} \rightarrow \infty$ as $y \rightarrow H$, it is convenient to introduce the dimensionless quantity χ as

$$\chi = \frac{V}{H_0 \epsilon_{eq}}. \quad (17)$$

Substituting (8), (11), (16) and (17) into (13) yields

$$\chi = \frac{\sqrt{3}}{2(p+1)} \sqrt{1 - t^2 Y^2 \left(\frac{\chi^m + \Omega^m}{\chi^m + t\Omega^m} \right)^2} \quad (18)$$

where

$$p = \frac{H_0}{H} - 1 \quad \text{and} \quad \Omega = \frac{V}{H_0 \epsilon_0}. \quad (19)$$

Equation 18 determines χ as a function of p and Y in implicit form. The plastic work rate is given by

$$W = \sigma_{eq} \epsilon_{eq}. \quad (20)$$

It follows from (8), (11), (17) and (19) that $\sigma_{eq}/\sigma_0 = (\chi^m + t\Omega^m)/(\chi^m + \Omega^m)$. This equation, (17) and (20) combine to give

$$W = \frac{V\sigma_0}{H_0\chi} \left(\frac{\chi^m + t\Omega^m}{\chi^m + \Omega^m} \right). \quad (21)$$

The constant C involved in (14) is determined from the boundary condition (10). Using (14) and (16) this condition can be rewritten as

$$C = 2 \int_0^1 \left[\int_1^Y \cot(2\varphi) d\upsilon \right] dY \quad (22)$$

where υ is a dummy variable of integration. Using (8), (11) and (19) it is possible to find from (15) that

$$\cot(2\varphi) = \frac{\sqrt{3 - 4\chi^2(p+1)^2}}{2\chi(p+1)}. \quad (23)$$

Here χ should be eliminated by means of the solution of Equation 18 in which Y should be replaced with υ . The boundary condition (9) has no effect on the temperature field.

3 | TEMPERATURE FIELD

Since $\chi = 0$ at $Y = 1$, it is evident from (21) that $W \rightarrow \infty$ as $Y \rightarrow 1$. On the other hand, W is involved in the heat conduction equation. Therefore, this equation contains a singular term. This greatly adds to the difficulties of a solution. In particular, standard finite element packages are not capable of calculating the correct distribution of W in the vicinity of maximum friction surfaces.^[20]

By assumption, the temperature is independent of x . This assumption has no effect on the general asymptotic singular behavior of the plastic work rate in the vicinity of the friction surface. Moreover, it is generally accepted in metal forming applications that the variation of temperature in the x -direction is negligible.^[21]

If the temperature is independent of x then the heat conduction equation can be written as

$$\frac{dT}{dt} = \frac{\lambda}{c_v \rho} \frac{\partial^2 T}{\partial y^2} + \frac{\beta}{c_v \rho} W. \quad (24)$$

Here T is the temperature, λ is the thermal conductivity, c_v is the specific heat, ρ is the density, d/dt denotes the convected derivative. The factor β determines the portion of plastic work converted into heat. It is seen from (24) that the temperature is directly proportional to β . Therefore, simple scaling supplies the solution for any value of β if the solution for $\beta = 1$ is known. For this reason, in what follows it is assumed that $\beta = 1$. It is convenient to introduce the dimensionless temperature τ as

$$\tau = \frac{T - T_0}{T_0} \quad (25)$$

where T_0 is the initial temperature of the strip. It is assumed that T_0 is constant. Since $dH/dt = -V$ and $dT/dt = \partial T/\partial t$ in the Lagrangian coordinates, Equation 24 can be rewritten using (16), (19), (21) and (25) as

$$\frac{\partial \tau}{\partial p} = a \frac{\partial^2 \tau}{\partial Y^2} + \frac{\sqrt{3}bt}{2\chi(p+1)^2} \left(\frac{\chi^m + \Omega^m}{\chi^m + t\Omega^m} \right) \quad (26)$$

where

$$a = \frac{\lambda}{V H_0 c_v \rho} \quad \text{and} \quad b = \frac{2\sigma_0}{\sqrt{3}T_0 c_v \rho}. \quad (27)$$

Equation 26 is the standard nonhomogeneous heat conduction equation. Therefore, this equation supplemented with this or that set of standard initial and boundary conditions can be efficiently solved using the Green's function.^[18] However, a difficulty is that the last term in (26) is singular as $Y \rightarrow 1$. In order to accurately find the temperature field in the vicinity of the friction surface, it is necessary to carry out asymptotic analysis of the solution as $Y \rightarrow 1$.

The solution of Equation 18 in the vicinity of the friction surface is represented as

$$\chi = \frac{\sqrt{3}}{2(p+1)} \sqrt{1-Y^2} \left[1 - \frac{(t-1)}{t(p+1)^m} \left(\frac{\sqrt{3}}{2\Omega} \right)^m (1-Y^2)^{\frac{m}{2}-1} + o(1-Y^2)^{\left(\frac{m}{2}-1\right)} \right] \quad (28)$$

as $Y \rightarrow 1$ if $m > 2$ and

$$\chi = \frac{\sqrt{3}}{2(p+1)} \sqrt{1-Y^2} \left[\left(1 + \frac{3(t-1)}{2t(1+p)^2\Omega^2} \right)^{-1/2} + O(1-Y^2) \right] \quad (29)$$

as $Y \rightarrow 1$ if $m = 2$. Using (28) and (29) the last term in Equation 26 is represented as

$$\frac{\sqrt{3}bt}{2(p+1)^2} \left(\frac{\chi^m + \Omega^m}{\chi^m + t\Omega^m} \right) \frac{1}{\chi} = \frac{b}{(p+1)\sqrt{1-Y^2}} \left[1 + \frac{(t-1)}{t(p+1)^m} \left(\frac{\sqrt{3}}{2\Omega} \right)^m (1-Y^2)^{\frac{m}{2}-1} \right] + o\left(\frac{1}{\sqrt{1-Y}} \right) \quad (30)$$

as $Y \rightarrow 1$ if $m > 2$ and

$$\frac{\sqrt{3}bt}{2(p+1)^2} \left(\frac{\chi^m + \Omega^m}{\chi^m + t\Omega^m} \right) \frac{1}{\chi} = \frac{b}{(p+1)\sqrt{1-Y^2}} \sqrt{1 + \frac{3(t-1)}{2t(1+p)^2\Omega^2}} + O(\sqrt{1-Y^2}) \quad (31)$$

as $Y \rightarrow 1$ if $m = 2$. It is seen from (30) and (31) that the last term in (26) involves one singular term of the order $O(1/\sqrt{1-Y})$ as $Y \rightarrow 1$ if $m = 2$ and $m \geq 3$ and two singular terms, one is of the order $O(1/\sqrt{1-Y})$ and the other of the order $O[(1-Y)^{(m-3)/2}]$ as $Y \rightarrow 1$, if $2 < m < 3$. In order to numerically solve Equation 26 with high accuracy, it is desirable to eliminate these singular terms. To this end, it is convenient to introduce the new function u as

$$u = \tau + D_1(p) \left(\sqrt{1-Y^2} + Y \sin^{-1} Y - 1 \right) + D_2(p) (1-Y^2)^{\frac{m+1}{2}} \quad (32)$$

where

$$\begin{aligned} m = 2: D_1(p) &= \frac{b}{(p+1)a} \sqrt{1 + \frac{3(t-1)}{2t(1+p)^2\Omega^2}}, \quad D_2(p) = 0, \\ 2 < m < 3: D_1(p) &= \frac{b}{(p+1)a}, \quad D_2(p) = \frac{b}{a(p+1)^{m+1}} \frac{t-1}{t(m^2-1)} \left(\frac{\sqrt{3}}{2\Omega} \right)^m, \\ m \geq 3: D_1(p) &= \frac{b}{(p+1)a}, \quad D_2(p) = 0. \end{aligned}$$

4 | ILLUSTRATIVE EXAMPLE

For definiteness, it is assumed that $m \geq 3$. Then, Equation 32 becomes

$$u = \tau + \frac{b}{a(p+1)} \left(\sqrt{1-Y^2} + Y \sin^{-1} Y - 1 \right). \quad (33)$$

Substituting (33) into (26) gives

$$\frac{\partial u}{\partial p} = a \frac{\partial^2 u}{\partial Y^2} - \frac{b}{a(p+1)^2} \left(\sqrt{1-Y^2} + Y \sin^{-1} Y - 1 \right) - \frac{b}{(p+1)\sqrt{1-Y^2}} + \frac{\sqrt{3}bt}{2\chi(p+1)^2} \left(\frac{\chi^m + \Omega^m}{\chi^m + t\Omega^m} \right). \quad (34)$$

Using (33) transforms the initial condition $\tau = 0$ at $p = 0$ to

$$u = \frac{b}{a} \left(\sqrt{1-Y^2} + Y \sin^{-1} Y - 1 \right) \quad (35)$$

at $p = 0$. The heat flux through the axis of symmetry $y = 0$ vanishes. Therefore, $\partial\tau/\partial y = 0$ at $y = 0$. Using (16) and (33) transforms this boundary condition to

$$\frac{\partial u}{\partial Y} = 0 \quad (36)$$

for $Y = 0$. It is assumed that the external heat flux through the friction surface vanishes. Then, it follows from (12) that

$$\lambda \frac{\partial T}{\partial y} = \frac{\sigma_s}{\sqrt{3}} v_x \quad (37)$$

for $Y = 1$. The right hand side of this equation is the heat generated by friction. The value of v_x at $Y = 1$ (or $y = H$) is found from Equation 14. In particular, using (19)

$$v_x = V [l_0 X (p+1) + C] \quad (38)$$

at $Y = 1$. Here $X = x/L_0$ and $l_0 = L_0/H_0$. It follows from (16), (19), (25) and (28) that

$$\frac{\partial T}{\partial y} = \frac{T_0}{H_0} \left[(p+1) \frac{\partial u}{\partial Y} - \frac{b}{a} \sin^{-1} Y \right]. \quad (39)$$

Substituting (38) and (39) into (37) and using (27) yields

$$\frac{\partial u}{\partial Y} = \frac{bt}{2a(p+1)} [l_0 X(p+1) + C] + \frac{\pi b}{2a(p+1)} \quad (40)$$

for $Y = 1$. It is evident that this boundary condition is not compatible with the assumption that the temperature is independent of x . However, it is generally accepted in metal forming applications that the variation of the temperature in the x -direction is negligible and the right hand side of (37) is calculated at one point of the friction surface (see, for example, [21]). A more reasonable assumption is to replace the right hand side of (37) with its average value.^[22] The latter assumption is adopted in the present paper. The approximate solution (14) is not valid in the vicinity of the axis $x = 0$. In the exact solution, a sticking region where $v_x = 0$ occurs near this axis and no heat is generated by friction. It is seen from (38) that $v_x = 0$ at $X = X_0$ where

$$X_0 = -C(1+p)^{-1}l_0^{-1}. \quad (41)$$

Then, using (16) the boundary condition (37) is approximated as

$$\frac{\partial u}{\partial Y} = \frac{bt}{2a(p+1)(L/L_0 - X_0)} \int_{X_0}^{L/L_0} [l_0 X(p+1) + C] dX + \frac{\pi b}{2a(p+1)} = \frac{bt}{2a} \left[\frac{l_0}{2} (p+1 + X_0) + \frac{C}{(p+1)} \right] + \frac{\pi b}{2a(p+1)} \quad (42)$$

for $Y = 1$. It has been taken into account here that $L/L_0 = p+1$. This equation results from the equation of incompressibility in the form $LH = L_0H_0$ and Equation 19.

Equation 34 along with the initial condition (35) and the boundary conditions (36) and (42) comprise the second initial/boundary value problem for the nonhomogeneous heat conduction equation. Its solution is given by [18]

$$u(Y, p) = \frac{b}{a} \int_0^1 \left(\sqrt{1 - \xi^2} + \xi \sin^{-1} \xi - 1 \right) G(Y, \xi, p) d\xi + \int_0^p \int_0^1 S(\xi, \theta) G(Y, \xi, p - \theta) d\xi d\theta + \int_0^p B(\theta) G(Y, 1, p - \theta) d\theta \quad (43)$$

where

$$S(\xi, \theta) = -\frac{b}{a(\theta+1)^2} \left(\sqrt{1 - \xi^2} + \xi \sin^{-1} \xi - 1 \right) - \frac{b}{(\theta+1)\sqrt{1 - \xi^2}} + \frac{\sqrt{3}bt}{2\chi(\theta+1)^2} \left(\frac{\chi^m + \Omega^m}{\chi^m + t\Omega^m} \right),$$

$$B(\theta) = \frac{bt}{2} \left[\frac{l_0}{2} (\theta+1 + X_0) + \frac{C}{(\theta+1)} \right] + \frac{\pi b}{2(\theta+1)}$$

and

$$G(Y, \xi, p) = 1 + 2 \sum_{n=1}^{\infty} \cos(n\pi Y) \cos(n\pi \xi) \exp(-an^2 \pi^2 p)$$

$$= \frac{1}{2\sqrt{\pi ap}} \sum_{n=-\infty}^{n=\infty} \left\{ \exp \left[-\frac{(Y - \xi + 2n)^2}{4ap} \right] + \exp \left[-\frac{(Y + \xi + 2n)^2}{4ap} \right] \right\}.$$

It is understood here that χ is found as a function of ξ and θ from (18) in which Y should be replaced with ξ and p with θ .

The integrals in (43) have been evaluated numerically. For all of the calculations, it has been assumed that $\lambda = 36 \text{ W} \cdot \text{mm}^{-1} \cdot \text{K}^{-1}$, $c_v \rho = 3.77 \text{ J} \cdot \text{mm}^{-3} \cdot \text{K}^{-1}$ and $\sigma_0 = 400 \text{ MPa}$. These physical properties correspond to carbon steel AISI 1015.^[23] It has been also assumed that $T_0 = 20^\circ\text{C}$, $V = 5 \text{ mm} \cdot \text{s}^{-1}$ and $V = 50 \text{ mm} \cdot \text{s}^{-1}$, $m = 3.5$, $t = 3$, $H_0 = 9 \text{ mm}$ and $L_0 = 45 \text{ mm}$. The values of V chosen are typical in forging.^[24] A spatio temporal representation of the solution at $\Omega = 1$ is depicted in Figure 2a for $V = 5 \text{ mm} \cdot \text{s}^{-1}$ and in Figure 2b for $V = 50 \text{ mm} \cdot \text{s}^{-1}$. It is seen from these figures that the higher speed results in higher temperature. It has been checked that the solution is not sensitive to the values of Ω and m in the ranges $0.7 \leq \Omega \leq 10$ and $3 \leq m \leq 10$. This is in accordance with the conclusion based on general considerations made in [14]. It is seen from Figure 2 that the gradient of τ is very high in the vicinity of the friction surface if the value of H/H_0 is small enough.

In order to better illustrate this feature of the solution, the through thickness distribution of the dimensionless temperature in the Eulerian coordinates at several values of H/H_0 is depicted in Figure 3a for $V = 5 \text{ mm} \cdot \text{s}^{-1}$ and in Figure 3b for

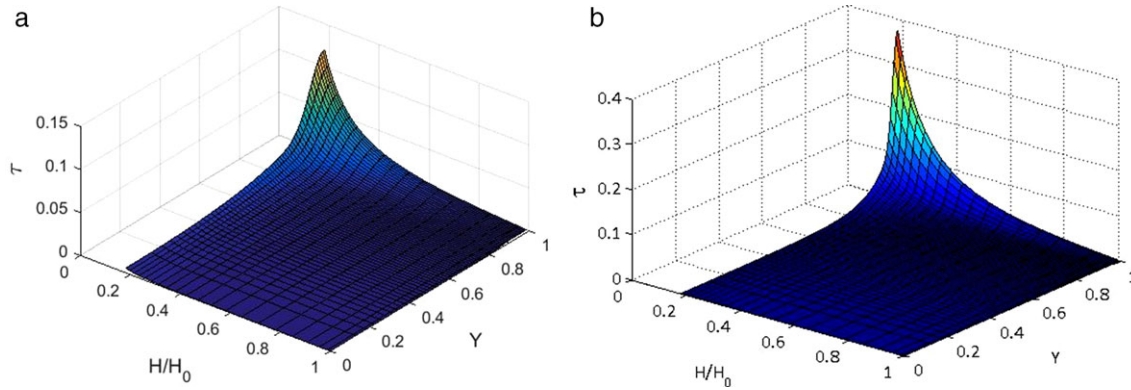


FIGURE 2 Spatio temporal distribution of the dimensionless temperature in the Lagrangian coordinates: (a) at $V = 5 \text{ mm} \cdot \text{s}^{-1}$ and (b) at $V = 50 \text{ mm} \cdot \text{s}^{-1}$

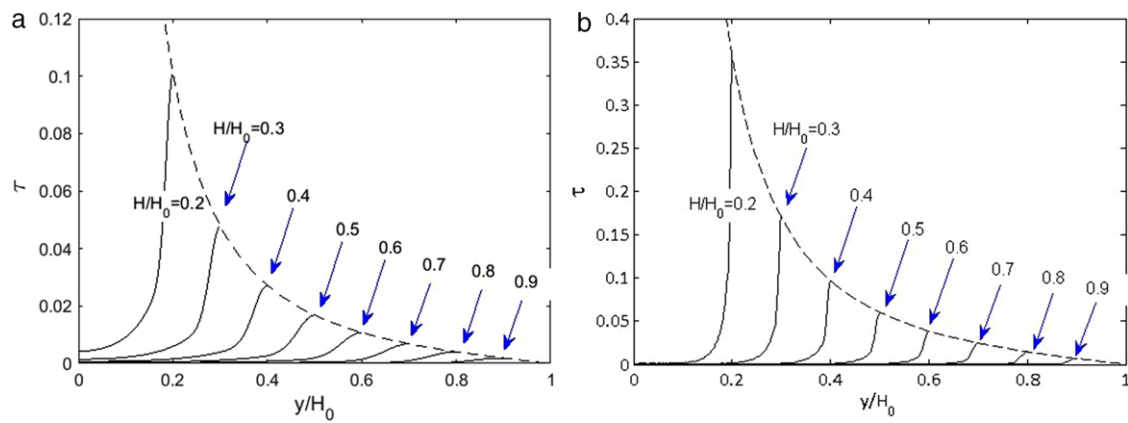


FIGURE 3 Through thickness distribution of the dimensionless temperature in the Eulerian coordinates: (a) at $V = 5 \text{ mm} \cdot \text{s}^{-1}$ and several values of H/H_0 and (b) at $V = 50 \text{ mm} \cdot \text{s}^{-1}$ and several values of H/H_0

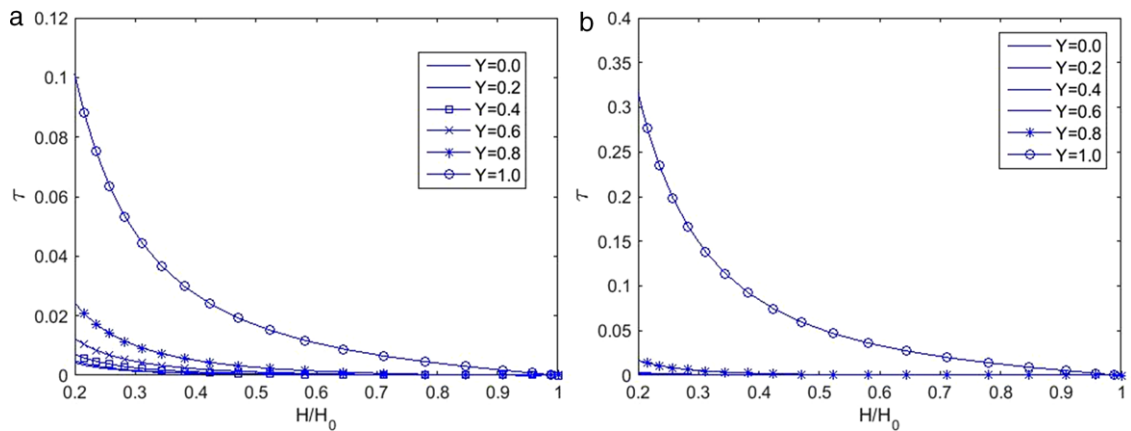


FIGURE 4 Variation of the dimensionless temperature with H/H_0 : (a) at $V = 5 \text{ mm} \cdot \text{s}^{-1}$ and several values of Y and (b) at $V = 50 \text{ mm} \cdot \text{s}^{-1}$ and several values of Y

$V = 50 \text{ mm} \cdot \text{s}^{-1}$. It is seen from these figures that the gradient of temperature is high near the friction surface in the range $H/H_0 < 0.4$ (approximately) if $V = 5 \text{ mm} \cdot \text{s}^{-1}$ and in the range $H/H_0 < 0.6$ (approximately) if $V = 50 \text{ mm} \cdot \text{s}^{-1}$. In experimental works, temperature-versus-time data at several points are usually presented (see, for example, [25]). Typical plots of temperature versus H/H_0 are shown in Figure 4a for $V = 5 \text{ mm} \cdot \text{s}^{-1}$ and in Figure 4b for $V = 50 \text{ mm} \cdot \text{s}^{-1}$.

5 | CONCLUSIONS

The interface between tool and workpiece in metal forming processes is crucial to both friction and heat transfer.^[26] The temperature distribution near the interface affects material properties in a narrow sub-surface layer.^[5] Usually, commercial finite element packages are used to study heat transfer in metal forming processes (for example, [27–29] among many others). A conventional method for increasing the accuracy of finite element predictions of the temperature field near friction interfaces is to use a fine mesh near the interface (see, for example, [29]). However, for a number of material models widely used for the modelling of metal forming processes the plastic work rate is described by non-differentiable functions in the vicinity of maximum friction surfaces. To accurately predict the temperature field in such cases, it is desirable to account for the exact asymptotic expansion of solutions near the friction surface. The present paper develops such an approach for the process of strip upsetting between two parallel plates. The viscoplastic model with a saturation stress has been adopted. Solutions based on this model predict very high gradients of the equivalent strain rate near maximum friction surfaces, which is in agreement with experiment. It has been shown in the present paper that this model predicts a high gradient of the temperature near the friction surface as well, which is also in agreement with experiment.

The solution presented is a generalization of the solution developed in [22] where a model of rigid perfectly plastic material was considered. In the same manner, the solution can be generalized on other models for which the solution for stress and velocity is available.^[30–33]

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